A Differtation concerning the Figure of the Earth, by the Reverend John Theophilus Defaguliers, LL. D. F. R. S.

THAT the Earth (that is, our Terraqueous Ha-
bitation) is of a fpherical Figure, or nearly fuch,
has been prov'd fo often, and by fo many unanfwer-
able Arguments, that to repeat them here muft needs
be tedious to this learned Society. But, as a little Va-
riation from a true Sphere (befides the Irregularity of
high Hills and deep Vallies) does not hinder us from
calling the Earth a Globe; fo, to determine what that
Variation may be, fince modern Philofophers are di-
vided about it, may be a Subject not ungrateful at this
Time.

Monfieur *Caffini* fays: " That the Earth is an ob-
" long Spheroid, higher at the Poles than the Æquator,
" making the Axis longer than a Diameter of the Æqua-
" tor about thirteen *French* Leagues, which he de-
" duces from comparing his Father's Meafures of the
" Meridian, from *Paris* to the *Pyrenæan* Moun-
" tains, with thofe of Monfieur *Picard*; of which an
" Account may be feen in the Memoirs of the Royal
" Academy for 1713. But having afterwards con-
" tinued the Meridian, which is drawn through *France*,
" from *Paris* to *Dunkerque*, he ftill draws Confe-
" quences to prove the Earth an oblong Spheroid;
" but then makes the Axis exceed the Æquatorial
" Diameter 34 Leagues.

G g " Sir

" Sir *Ifaac Newton* makes the Earth higher at the
" Æquator, and, confequently, flatted towards the
" Poles, reckoning its Æquatorial Diameter 34 *Englifh*
" Miles longer than the Axis; which he proves from
" the Principles of Gravity, and the Centrifugal Force
" that arifes from the Diurnal Rotation of the Earth;
" and, to confirm this, mentions feveral Experiments
" on Pendulums, which have been made fhorter, to
" fwing Seconds, near the Æquator, than in greater
" Latitudes."

Thefe are the two Opinions which have divided
Philofophers, and which we propofe to examine here.

Monfieur *Caffini*, taking the Meafures above-men
tion'd to be exact enough, not only to determine the
Magnitude of a Degree of the Earth, correfponding
with a Degree of the great Circle of the Heavens, but
alfo to fhew the Difference in the Degrees of the Earth ;
(reckoning thofe, that were meafured in the South of
France, to exceed thofe towards the North, by a
certain Number of Toifes and Feet) demonftrates, that
if the Degrees of the Earth are longer towards the
Æquator than the Poles, the Plane of the Meridian
muft be an Ellipfe, whofe long Axis is that of the
Earth. Here follows his firft Demonftration. [*See the*
French *Memoirs for the Year* 1713]

" Let B D C R * be an Ellipfe that reprefents a
" Meridian of the Earth, whofe Poles B and C are at
" Ends of the great Axis B C, and whofe Foci E and
" F are taken at Pleafure. *Now, to divide this*
" *Ellipfe into Degrees, that is, to find feveral*
" *Points* H, I, V, *fuch, that the Diftance, from the*
" *Pole to the Zenith, of every one of them, fhall*
" *be of any given Number of Degrees.*

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* Fig. 1.

" From

" From E, one of the Foci of the Ellipſe, draw the
" Line E T, ſo that it may, with the Axis B C,
" make the Angle B E T equal to the Diſtance given
" from the Pole to the Zenith. From the other Focus
" F, with the Diſtance B C equal to the Axis, draw
" an Arc, to cut the Line E T at T. I ſay, that the
" Line F T, drawn from the Point T to the Focus
" F, will cut the Ellipſe at the Point H; which Point
" is ſuch, that the Diſtance of the Pole, from its
" Zenith, contains the given Number of Degrees.

"DEMONSTRATION.

" From the Point H, raiſe H Z, perpendicular to
" the Ellipſe which will paſs through the Zenith Z ;
" and, being produc'd inwards, will meet the Axis of
" the Earth at O, and (by the Property of the Ellipſe)
" divide the Angle E H F into two equal Parts. From
" the Point H, draw likewiſe H P, parallel to the Axis
" B C, and directed to the Pole P, ſuppos'd at an infi-
" nite Diſtance. The Angle P H Z, or P O Z, mea-
" ſures the Diſtance, from the Pole to the Zenith, of
" an Inhabitant dwelling upon the Earth at the Point
" H. F T is equal to the Axis B C, by Conſtruction;
" but, by the Property of the Ellipſe, B C is equal to
" E H *plus* H F ; taking away from both F H, which
" is common, E H will remain equal to H T. The
" Angles E T H, T E H, will therefore be equal,
" and, conſequently, each of them will be half of the
" external Angle E H F ; but the Angle E H O is
" likewiſe equal to half of the Angle E H F ; there-
" fore the Angles T E H, E H O, will be equal to
" one another; and, conſequently, the Lines E T and
" H O will be parallel to one another ; and the Angle
" P O Z, which meaſures the Diſtance from the Pole

" to

" to the Zenith of the Point H, will be equal to the
" Angle B E T, which was, by Conftruction, taken
" equal to the given Diftance of the Pole from the
" Zenith ; *which was to be demonftrated.*

" Now, if the Proportion of the longeft Diameter
" of the Ellipfe B C to E F, the Diftance of the Foci,
" be taken at Pleafure, one may by Calculation
" find all the Points of the Ellipfe as H, to deter-
" mine the Degrees by this Analogy.

" *As F T, or B C :*

" *Is to E F : :*

" *So is the Sine of the Angle P E T (the given Di-*
" *ftance from the Pole to the Zenith) :*

" *To the Sine of the Angle E T F, or T E H :*
" whofe Quantity will confequently be known. This
" Angle T E H being added to the Angle P E T, the
" given Diftance from the Pole to the Zenith of the
" Point H, will give the Quantity of the Angle B E H,
" which a Line drawn from the Focus to H, the Point
" requir'd, makes with the Axis of the Ellipfe.

" Then in the Triangle E H F, whofe Side E F is
" known, as well as the Angle E H F, which is the
" Double of the Angle T E H, and the Angle F E H
" Supplement of the Angle B E H ; one fhall have the
" Length of the Side E H, known in Parts of the Axis
" B C.

" After the fame Manner, may be found the Angles
" B E I, B E V, *&c.* and the Length of the Lines E I,
" E V, to determine the Diftance, from the Pole to the
" Zenith, of all the Degrees of the Circumference of
" the Earth ; and in the rectilinear Triangles H E I,
" I E V, whofe Sides H E, E I, E V, are known as
" well as the Angles comprehended between the Sides
" H E, E I, I E, E V, which are the Differences of
" the Angles B E H, B E I, B E V, determin'd above ;

" one

" one fhall find the Length of the Chords H I, I V,
" comprehended between each Degree.

Monf. *Caffini*, in the Memoirs for the Year 1718, re-
peats the fame Demonftration ; except that, before it, he
fhews, that if feveral Points be taken upon a Terreftrial
Meridian, on the Surface of an Elliptick Earth, as G, H,
I, K,* in fuch Manner, that their refpective Zeniths Z, L,
M, N, are diftant from one another, an equal Number
of Degrees meafur'd in a Celeftial Meridian. The Lines
Z G, L H, M I, N K (which are perpendicular to the
Ellipfe) being produced, will meet in the Points O, R,
and S, making equal Angles; but as thofe angular Points
are not equally diftant from the Curve of the Ellipfe,
that Elliptic Arc muft be the longeft whofe angular
Point is fartheft off. Now, by the former Demonftra-
tion, it appears, that thofe Arcs, which are taken neareft
to the leffer Axis, will have their angular Points farther
remov'd, &c.

If Monf. *Caffini's* Meafures of Terreftrial Degrees,
decreafing from the Æquator towards the Pole, were
grounded on Obfervations liable to no Error, he wou'd
have fully prov'd his Figure of the Earth. But fince
thofe Meafures (however accurately taken) are not built
upon a mathematical Certainty, his Premifes may be
call'd in Queftion, and his Conclufion, tho' mathema-
tically drawn from thefe Premifes, is only probable.

Now therefore, if I can fhew from undoubted *Phæ-
nomena*, that his Conclufion will lead to an Abfurdity,
his Meafures muft be falfe ; becaufe his Reafoning
from them is juft. This I fhall endeavour to do firft,
which will difprove his Figure of the Earth ; and after-
wards endeavour to point out fome of the Errors which

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* Fig. II.

I fuppofe to have occafion'd the Miftake in the Mea-
fures.

Monf. *Caffini*, as well as the *Englifh* Aftronomers,
believes that the Earth makes one Revolution about its
Axis, once in 23 Hours 56', becaufe in that Time, the
Plane of any Meridian returns to the fame fix'd Star
from which it had departed.

Let * H be taken in any Parallel of Latitude, as for
Example, in the Latitude of 51 ° 46', a Plumb Line,
L H, will be perpendicular to the Curve B H, at H, and
produced pafs thro' the Zenith of the Point H, if the
Earth had no Diurnal Rotation; but fince the Earth
moves round its Axis, all Bodies upon its Surface, en-
deavour to fly from the Axis of their Motion with a
Force proportionable to their Diftance from it in a Di-
rection along the Plane of that Parallel, in which they
are. Let that Force (explain'd by Monf. *Huygens*, and
called a Centrifugal Force) be reprefented by the Line
H *l*, or its Equal and Parallel L *h*; now a Plummet
placed at L, if the Earth ftood ftill, would defcend in
the Line L H, but as it is at the fame Time acted upon
by the Force H *l* in the Direction L *h*, it will move
in the Direction L *l* Diagonal of the Parallelogram H *l*,
according to the known Laws of Mechanicks; and the
Plumb Line L H, inftead of being perpendicular to the
Curve at H, will in the Latitude 51 ° 46' make an An-
gle of 5' with H L. This Angle will be lefs towards
the Poles, till at the very Pole it quite vanifhes, as it
alfo does at the Æquator. *The Demonftration of the
Quantity of this Angle, I fhall give before I end this
Differtation.* Now fince there is no fuch Angle ob-
ferv'd, but in all Water Levels we find the Plumb
Line always perpendicular to the Line of Level, the
<div align="right">Surface</div>

Surface of the Earth muſt be depreſs'd towards G, and
riſe farther from the Axis towards I, in order to become
perpendicular (that is, to have its Tangent perpendicular)
to the Line L *l*, in which we have ſhewn that the
Plumb Line muſt deſcend.

If there is any Body ſo fond of Monſieur *Caſſini*'s
Hypotheſis, as to deny the Diurnal Motion of the
Earth for the ſake of it, I hope they will be convinc'd,
when I ſhew the Meaſures, upon which it is founded, to
be inſufficient for determining the different Lengths of
the Degrees of a Terreſtrial Meridian.

But here I would not be thought to endeavour to
leſſen the Praiſe due to the Gentlemen of the *Royal
Academy*, for carrying on a Meridian the whole Length
of *France*, from *Dunkerque* thro' the Royal Obſer-
vatory at *Paris*, quite to the *Pyrenæan* Mountains
on the Borders of *Spain*. Aſtronomy and Geography
are doubtleſs much indebted to the Encouragement
given by the *French* Government, and to the Care of
their Mathematicians, who have omitted no proper
Method for drawing their Meridian, and correcting it as
they went on. So many Obſervations of the riſing and
ſetting Sun, ſo many equal Altitudes of the ſame Stars
accurately taken, ſo many Digreſſions of Stars, ſo many
other Obſervations made with the Teleſcope and good
Pendulum-Clocks — all compar'd together, for the true
ſettling of the direct Way of this famous Meridian, leave
no Doubt but that it is as perfect as the Nature of the
Thing is capable of. And, certainly, by the Help of
this Line, and the ſeveral Triangles made uſe of for
carrying it on, a better Map of *France* is made,
than has ever been of any Country before : Nay, be-
ſides, I believe we may, at a Medium, very well re-
ceive their Number of 57060, or 57061 Toiſes, for
the Meaſure of a Degree of a Meridian of the Earth,
one

one with another. But to fay, that thofe Gentlemen cou'd obferve the Latitude fo nicely, as to find a Difference in the Length of the Terreftrial Degrees, and that only of eleven or twelve Toifes, (when they made it the leaft) or of thirty-one Toifes, (when they made it the moft) is attributing to them an Exactnefs, fo far beyond the Nature of the Inftruments which they made ufe of, that it wou'd be rather a Difpraife than a Commendation to infift upon it.

For in the firft Place, the Inftrument, with which they took Obfervations for the Latitude at the two Ends of their Meridian, was a ten Foot Sector (which was worfe than that which Monf. *Picard* had made Ufe of before, becaufe the Telefcope of his Sector was of ten Foot, whereas Monf. *Caffini's* was but of three Foot, tho' applied to the ten Foot Sector) where the two hundreth Part of an Inch anfwers to eight Seconds of a Degree : Now the two hundredth Part of an Inch, being one of the leaft vifible Parts that we can fee in a divided Line ; they could not take an Angle nearer than that; nay, their Inftrument, according to their own Defcription of it, was divided but to every twenty Seconds. Now they allow, that fixteen Toifes, upon the Surface of the Earth, anfwer to one Second in the Heavens ; and they don't pretend to have taken an Obfervation nearer than to about three Seconds, which therefore cannot determine a Difference lefs than forty eight Toifes ; whereas the Degrees are only fuppofed to decreafe at moft, thirty one Toifes each, from *Collioure* to *Dunkerque.* But an Error of eight Seconds wou'd make a Difference of one hundred and twenty eight Toifes, on the Surface of the Earth ; above ten Times greater than the Difference of Degrees in the firft Suppofition, and four Times greater than that Difference in the laft. Befides, the Latitude was not obferv'd in the intermediate Places

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between *Paris*, and *Collioure*, with the abovemention'd
Inftrument of ten Foot Radius.; but they made Ufe
of a Quadrant, whofe Radius was only thirty nine
Inches, and fometimes an Octant of three Foot Ra-
dius. Nay, they fay themfelves, in their Account,
that it is not the Obfervations made at the Ends of
the Meridian, that we are to deduce the Difference of
the Length of a Degree from, but the Altitudes taken
at feveral Places between the Extreams; and, if we
grant, that they can take an Angle very well, to four
or five Seconds, with the great Inftrument, they can-
not come nearer than twelve or fifteen Seconds, with the
Quadrant or Octant, which we muft depend upon for
the Difference of the Meafure of Degrees : So that upon
the whole, we are to determine a Length of thirty one
Toifes, by an Inftrument which is liable to err above
two hundred.

If any Confequences of this Kind cou'd be drawn
from actual meafuring, a Degree of Latitude fhou'd be
meafur'd at the Æquator, and a Degree of Longitude
likewife meafur'd there ; and a Degree very northerly,
as for Example, a whole Degree might be actually
meafur'd upon the *Baltick* Sea, when frozen, in the
Latitude of fixty Degrees.——— There, according to
Monf. *Caffini*'s laft Suppofition, a Degree wou'd be of
56653 Toifes, whereas, at the Æquator, it wou'd be
of 58019 Toifes, the Difference being 1364 Toifes,
about the two and fortieth Part of a Degree, which
muft be fenfible ; and likewife the Degree of Longi-
tude wou'd, according to him, be of 56817 Toifes, lefs
by 1202, or the forty eighth Part, than a Degree of
Latitude at the fame Place.

But here it may be objected, that tho' the Latitude
was not taken with the ten Foot Sector, in the inter-
mediate Places between *Paris* and *Collioure*, yet the

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Latitude was taken with that Inſtrument at *Dunkerque*,
Paris, and *Collioure*, and therefore the ſouthern Part
of the Meridian, containing 6° 18′ 56″ may be com-
par'd with the northern Part of it, which contains 2°
12′ 16″ ; and that the former appears to contain more
Toiſes, in Proportion to the Difference of Latitude at its
Extremities, than the latter. To this may be anſwer'd,
that, even in this Caſe, the Obſervations made cannot
be nice enough to determine the Difference of the Length
of Degrees ; but there is another Error, which might
conſiderably miſlead the *French* Gentlemen, and make
the Degrees appear longer in the South of *France* ; that
is, the Error in taking the true Height of ſeveral
Mountains in *Auvergne*, *Languedoc*, and among the
Pyrӕneans, For if they have allow'd too much for
the Air's Refraction (which, by the Obſervations of Tra-
vellers, is greater towards the northern Regions, and
diminiſhes as we go Southward) the Heights of thoſe
Mountains will be taken too little, and their Baſes con-
ſequently longer, which will make the Degrees appear
bigger than they are. Let A B C D, * for Example,
be a Mountain, as the Mountain of *Rodez*, in the La-
titude of 44° 21′, whoſe Height B D is 300 Toiſes,
and whoſe Sides A B and B C (ſuppoſ'd to make an
Angle of 26° 33′, with the Horizon) are found by
Trigonometry, to be of 670 8 Toiſes each ; if by a Mi-
ſtake, in taking the Height, it be ſuppoſ'd only equal
to E D, or 257 Toiſes, then the Lines A B and B C
will become E F and E G ; ſo that the Baſe A C, which
before was of 1200 Toiſes, will become equal to F G,
which will appear to be 1279,6 Toiſes, by *Eucl.* 47.
1. Now one ſuch Miſtake, in one Degree, will give
a Difference above twice as great as the ſuppoſ'd Diffe-
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* Fig. III.

rence of Degrees in that Latitude, which they make of
31 Toifes. And that there was a Miftake of this Kind
in taking the Height of that Mountain, I fhall fhew.

The Vapours, that generally float in the Air about
the Tops of high Hills, make it fo difficult to take their
Height exactly, that Experiments, made with the Ba-
rometer, will, by obferving the Fall of the *Mercury*,
fhew the Height nearer than any Thing elfe we know
of. There were, indeed, feveral Experiments made
with the Barometer,*where the Differences of the Height
of the *Mercury*, from the Heights at which it ftood at
the *Royal Obfervatory*, are faid to anfwer to fo many
Toifes ; but of nine Obfervations mention'd by Monf.
Caffini, there are not two where the Number of Toi-
fes, faid to correfpond to the Heights of the Barometer,
do agree together.

The firft Experiment of the Barometer there men-
tion'd, made at *Collicure*, was this, " At the Height
" of 11⅝ Toifes above the Sea, the Barometer was fet
" up, and the *Mercury* ftood 3⅞ Lines higher than at
" the *Royal Obfervatory* (in the Tower of the eaftern
" Hall) at the fame Time ; and therefore, fince that
" Tower is 44 Toifes higher than the Sea, 3⅞ Lines
" of *Mercury* muft anfwer to 32½ Toifes.

Now, reducing thefe Toifes to Feet, and dividing
by 3⅝ it will appear that an Height of 58,5 Feet will
anfwer to the Fall of one Line of *Mercury* in the Ba-
rometer. Let this be taken as the Standard, and the
other Obfervations be compar'd with it. This may be
done by the following Table, where the firft Column
fhews the Place where the Obfervation was made ; the
fecond, the Fall or Rife of *Mercury* at each Place ex-
prefs'd in Lines, or 12th Parts of a *French* Inch ; the

H h 2   third,

third, the Heights or Depths anfwering to thofe Lines of *Mercury*, which, in the Memoirs, are given in Toifes, but here reduced to Feet ; the fourth, the Number of Feet anfwering to one Line of *Mercury* in each Obfervation, which is the Quotient of the Feet in the third Column, divided by the Number of Lines in the Second.

Obfervations of the Ba^ro-meter made at	Lines of Mercury.	Said to corre-fpond with Feet.	The Fall of one Line of Mercury an-fwers to Feet.
I. Collioure - -	$03\frac{1}{2}$	195	58,5
II. The Tower of Maffane.	31	2382	76,8
III. Bugarac -	42	3636	86,5
IV. Rupeyroux -	30	2181	72,7
V. Rodez -	24	1647	68,6
VI. Rodez -	20	1425	71,25
VII. Courlande -	54	4812	89,1
VIII. Cofte - -	54	4890	92,4
IX. Clermont -	03	200	66,6

A Sight of this Table will convince any one, that thefe Obfervations are not to be depended upon for determining the Height of the Mountains in the South of *France* ; for the Differences are not fmall, fuch as might happen in making the Experiments ; but fuch as render the Obfervations ufelefs for the Purpofes above-mention'd. For Example, the firft and the feventh differ almoft $\frac{1}{3}$: And if 58,5 Feet were allow'd for the Fall of one Line of *Mercury* in the feventh Obfervation, inftead of 944 Feet, then the Mountain of *Cofte* would be but 3085 Feet, inftead of 4890. Nay, upon examining the Memoirs, I find that in feveral Obfervations the Number of Toifes, faid to correfpond to a certain Height of *Mercury*, are only anfwerable to the Height of the Mountain above the Level of the Sea found by Trigonometry, from which the Height of the *Royal Obfervatory*, above the Sea, is fubftracted ; though, by the Manner of the Expreffion, a curfory Reader would imagine

imagine, that the Number of Toifes nam'd, was always proportionable to the Fall of the *Mercury*, and think all the Experiments and Obfervations very accurately made, when they feem to agree fo well in every Refpect.

Now after all, I do not queftion but that the Height of the Barometer, might be as it is fet down in the Memoirs, and well enough obferv'd; but it was wrong to compare the Height of the *Mercury* in the South of *France*, with the Height that the *Mercury* was at in the Barometer of the *Royal Obfervatory* at the fame Time; for, at that great Diftance and Difference of Latitude, the Weather (and confequently the Preffure of the Air and Height of the Barometer at the fame Level) might very much vary.

Even when there is fair Weather all over *France*, it does not follow that the Barometer fhall ftand at the fame Height. Let us fuppofe, for Example, that a North Wind blows: Where-ever the Air is check'd by a Chain of Mountains that run Eaft and Weft, it will be accumulated over thofe Mountains, and confequently prefs more as its Columns are higher; which will make the *Mercury* rife higher than it wou'd do with the fame Wind, if there were no Mountains, or if they ran North and South.

The Way, to have made the Experiments with the Barometer exactly, wou'd have been to have obferv'd the Height of the *Mercury* at the Bottom and at the Top of the Mountain, and that with a Tube of a pretty large Bore (with a proportionably large Ciftern for the ftagnant *Mercury*) becaufe, in a fmall Tube, the *Mercury* will often ftick to the Sides, and rife irregularly, as it will alfo in inclin'd Barometers. Simple Barometers are the beft, and a magnifying Glafs may be made ufe

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of to obferve fmall Rifes or Falls, having two fine and well made Indices to the Tube.

Dr. *Edmund Halley*, our Royal Aftronomer, has, fome Years ago, given us, in the *Philofophical Tranfactions*, the Falls of *Mercury* in the Barometer, correfponding with the Heights to which the Barometer muft be carried to produce thofe Falls. The firft tenth Part of an Inch in the Fall of the *Mercury*, he makes to anfwer to an Height of 90 Feet ; the next tenth, to an Height fomething greater, and fo in Proportion, as the Air diminifhes in Denfity, according as we rife in Height. The Proportion of the firft Tenth of the *Mercury's* Fall, he has built upon the Comparifon of the different Specifick Gravities of Air and *Mercury*; and taking *Mercury* to be $13\frac{1}{2}$ Times heavier than Water, and Water (in cold Weather) to be 800 Times heavier than Air; it follows, that $13,5 \times 800$, will give 10800; which Number, if it be taken in Feet, and divided by 120 (the Number of the 10th of an Inch in a Foot) we fhall have 90 Feet anfwerable to the 10th of an Inch, and 75 Feet to a Line or the 12th Part of an Inch. As the Doctor's Tables may be of ufe to the Curious, I have inferted them here.

I. *A TABLE shewing the Altitude, to given Heights of the* Mercury.

Inches.			Feet.
30		0	
29		915	
28		1862	
27		2844	
26		3863	
25		4922	
20		10947	
15		18715	
10		29662	
5		48378	
1		91831	
0 . 5		110547	
0 . 25		129262	
0 . 1	29 Miles or	154000	
0 . 01	41 Miles or	216169	
0 . 001	53 Miles or	278338	

II. *A*

II. *A T A B L E shewing the Heights of the* Mercury, *at given Altitudes.*

Feet.		30	00	Inches and Tenths.
	0	30	00	
1000		28	91	
2000		27	86	
3000		26	85	
4000		25	87	
5000		24	93	
	1 Mile	24	67	
	2	20	29	
	3	16	68	
	4	13	72	
	5	11	28	
	10	4	24	
	15	1	60	
	20	0	95	
	25	0	23	
	30	0	08	
	40	0	012	

Now, as very few Mountains in the World are 3 Miles high, and, generally speaking, those that we look upon as high Hills, (except the *Andes*, and some others in *America*) are not much above a Mile high; we may, for finding the Height of Mountains, take a fix'd Number of Feet in Altitude to answer to every 10th or 12th of an Inch in the Fall of the *Mercury*; because 90 Feet are by Dr. *Halley* only taken for the first Tenth, and greater Heights for other Tenths, encreasing with the Fall of the *Mercury*. Therefore I wou'd propose another for a round Number, namely 96 Feet for every Tenth, and 80 Feet for every 12th of an Inch, very near the Number that I have found by my Calculation, which is as follows.   Fine

Fine *Mercury* (fuch as is made ufe of in Barome-
ters) is, generally fpeaking, 13⅓ times heavier than Wa-
ter ; and, I found fome brought from the *Eaft-Indies*,
to be 14 times heavier. I have found Air in Summer,
to be near 900 times lighter than Water; and 800
times in Winter ; therefore I take 850 at a Medium.
Now 850 × 13⅓ = 11606,6, which, divided by 120,
gives 96,7 Feet, for ₁⁄₁₀ of an Inch of *Mercury*, or 80,5
Feet for ₁⁄₁₂ of an Inch. This Number, taken invariable,
will, in taking the Height of feveral Hills, agree prettv
well with the Numbers that come out, when Dr. *Halley's*
Table is made ufe of ; and with the Experiment made
by the late Profeffor, Mr. *J. Cafwell*, who, having
taken the Height of *Snowdon* Hill in *Caernarvan-
fhire* very accurately, and finding it to be 3720 Feet
above the Level of the Sea, tried how much lower the
Mercury wou'd ftand in the Barometer upon that Hill,
than at the Level of the Sea, and obferv'd it to fubfide
3 9 Inches. I am fenfible that it will be alledged, that
the Air will be denfer than I may imagine on the Top
of high Hills, becaufe of the great Cold, fince they are
generally cover'd with Snow ; but then we are to con-
fider, that when we are got above a Mile higher than
the Level of the Sea, the incumbent Atmofphere has loft
almoft a 5th Part of its Weight * ; and therefore the Air
at the Top of the Hill, being fo much lefs prefs'd, will,
notwithftanding the intenfe Cold, be more rarified than
at the Bottom of the Hill.

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* *If the whole Air was reduced to the fame Denfity, as it is near the Level
of the Sea, the Atmofphere wou'd be but between five and fix Miles high ; where-
as, in its prefent State of gradual Rarefaction, it is above 50 Miles high, as we
find by feveral* Phænomena *of Meteors obferv'd to float in the Air, fo high at
leaft.*

Now if we go back to the Obfervations of the Baro-
meter, made by the Gentlemén that drew the Meri-
dian in *France*, we fhall find, that on the Mountain of
Rhodez, in the Latitude of 44°, 21′ the Barometer fell
24 Lines below the Level of that in the Obfervatory,
and they allow'd only 274½ Toifes to correfpond to that
Fall ; whereas, according to Dr. *Halley*'s Proportion of
a Tenth of an Inch for 90 Feet, they fhou'd have taken
300 Toifes ; and tho' the *Hypotenufes* A B, and B C,
* were taken longer than the bare Declivity of the
Mountain (which wou'd make the Error lefs than the
79 Toifes I mention'd above) yet if my Proportion be
made ufe of, *viz.* of 80 Feet for each Line of *Mer-
cury*, that will make the Mountain 320 Toifes, which,
being higher, will therefore fhew the Bafe to be yet
fhorter, and confequently the Error, at that Rate, will
be greater.

This Error (and fuch like, if any more were made)
will encreafe the Meafure of the 44th Degree of Lati-
tude on the Earth ; and, by obferving what was done
in the next Degree, we fhall find that that Degree was
taken too fhort. In the Latitude of 45°, 38′, the
Mountain of *Cofte* is made 815 Toifes high ; whereas
the 54 Lines of the falling *Mercury* in the Barometer,
faid to anfwer to that Height, will give but 705,6 Toi-
fes (which we will call 705,5) even according to my
Computation of 80 Feet to a Line, which is the greateft
Allowance. If we fuppofe this Mountain to rife in an
Angle of 26°, 33′, as we did that of *Rhodez*, the Sides
of the Mountain, or *Hypotenufes* A B, and B C, † will
be each equal to 1577,54 Toifes, and the whole Bafe
A C, to 2822 Toifes. Now, when the Height of this
Mountain is call'd 815 Toifes, the Bafe A D, or D C

(by

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* Fig. III.   † Fig. IV.

(*by Eucl.* 47. 1.) becomes only equal to F D or D G =
1350,7 Toiſes, and its Double, or FG the whole Baſe, will
be but 2701,4 Toiſes, leſs than the former by 120,6 Toi-
ſes. This Error is ſo great (ſo much more than the Diffe-
rence of 31 Toiſes for a Degree) that tho' I ſuppos'd the
Lines found by Trigonometry, which terminate at the
Top of the Mountain, to be much longer than the Hy-
potenuſe A B, yet there will be Error enough to make
the 45th Degree of Latitude appear much ſhorter than
it is. Suppoſing (becauſe of the Length of the Lines
A B, or the great Diſtance from which the Mountain
might be obſerv'd) that theſe Errors were four times
leſs than I made them; yet, at that Rate, one muſt add
near 20 Toiſes to the 44th Degree of Latitude, and take
away above 30 from the 45th Degree, which will make
the 44th of 57080 Toiſes, and the 45th, of only 57030;
and this will give a Difference of 50 Toiſes; ſo that if
an Angle can be taken to two or three Seconds, to
which 32, or 48 Toiſes, are ſaid to anſwer upon the
Surface of the Earth, ſuch a Difference might be viſi-
ble.—— And much more ſo, if other Errors of the ſame
Kind ſhou'd happen to have been made the ſame Way ;
or if thoſe Errors were nearer my firſt Suppoſition than
this laſt. Nay, tho' the 45th Degree of Latitude may
be 13 Toiſes bigger than the 44th, it might by this
Means appear to be conſiderably leſs.

Such a Miſtake might be the Occaſion of making the
Hypotheſis of the Earth being an oblong Spheroid, eſpe-
cially becauſe in this Hypotheſis, the Degrees differ
moſt in Length from one another about the 45th Degree;
and, when once an Hypotheſis is ſet on Foot, we are
too apt to draw in Circumſtances to confirm it ; tho',
perhaps, when examin'd impartially, they may rather
weaken, than ſtrengthen our Hypotheſis ; otherwiſe, the
Author of the Hiſtory of the *Royal Academy*, for the

Year

1713, wou'd not have alledg'd, that *the late Monſ.* Caſſini *obſerv'd* Jupiter *to be oval,* as a Proof of young Monſ. *Caſſini's* Hypotheſis ; becauſe *Jupiter* is oval the other Way, that is, an oblate Spheroid flatted at the Poles, as the ſaid late Monſ. *Caſſini* gave the Proportion of the Axis, to the Æquatorial Diameter, to be as 15 to 16.——— And our late ingenious Aſtronomer, the Reverend Mr. *Pound,* with a Teleſcope of 123 Foot *Focus,* and an excellent Micrometer, has given thoſe Proportions as 11 to 12. If a Proof is to be drawn from Analogy, or what is obſerv'd in other Planets, this muſt deſtroy Monſ. *Caſſini's* Hypotheſis, and confirm Sir *Iſaac Newton's.*

The Opinion of Dr. *Burnet* (the Theoriſt) quoted in the Memoirs, for the Year 1713, is but a very weak Argument in favour of Monſ. *Caſſini's* Hypotheſis, on Account of the Reaſon Dr. *Burnet* gave, to prove the Earth higher at the Poles, than the Æquator ; for he ſays, " That the Velocity of the Parts of the Earth, " in its Diurnal Rotation, being greater at the Æqua- " tor than towards the Poles, all the Water muſt be " driven towards the Æquatorial Regions; from " whence, being repelled by the Reſiſtance of the " Air, it muſt run off again towards the Poles ; and " ſo the Figure of the Water was lengthen'd out into an " oblong Spheroid, and conſequently the Cruſt of the " Earth over it did put on the ſame Figure, &c.

But why the Air ſhou'd reſiſt more towards the Æquator than the Poles, the Doctor did not give any Reaſon to ſhew ; and, if it had been ſo, the ſame Force, that drove the Water towards the Æquator, muſt have kept it there. The Doctor, in the latter Part of his Aſ-ſertion, forgot what he had ſaid in the former ; for the Water cou'd not run off towards the Poles, whilſt the Earth continued its Rotation with the ſame Velocity.

For

For if he had confider'd, he wou'd have found his Argument in other Words to be this. *Becaufe Bodies, that move in a Circle, always endeavour to recede from the Axis of their Motion; therefore the Water, by that Endeavour, comes nearer to the Axis of its Motion*; which is abfurd. But Dr. *Burnet*, afterwards, alter'd his Opinion, as I am credibly inform'd.

Having thus given my Reafons for difapproving of Monf. *Caffini*'s Opinion, concerning the Figure of the Earth; I come now to confider Sir *Ifaac Newton*'s, who makes it higher at the Æquator, than at the Poles; but before I enter upon it, I beg Leave to quote a Para. ragraph out of the Hiftory of the *Royal Academy* for 1713. Thefe are the Words of the Author. " Rea-
" fonings drawn from the different Lengths of a Pen-
" dulum in different Climates, or from the Inequality
" of the Centrifugal Force arifing from the Diurnal
' Motion of the Earth, are, perhaps, too nice to pro-
" duce a certain Conviction; nay, perhaps, we are not
" well enough affured of the Principles, and the
" Confequences may fometimes be different. And
" therefore it is evident, that the beft Way in this En-
" quiry, is only (as Monf. *Caffini* does) to make ufe
" of unqueftion'd Obfervations, which ferve directly
" to decide the Queftion.

That Monf. *Caffini* has not made ufe of *unqueftion'd Obfervations*, and the Meafures, he mentions, are not able to decide the Queftion, appears from what I have already faid. We muft therefore fhew, whether the Principles, from which Sir *Ifaac Newton* has deduced his Figure of the Earth, are fully prov'd or not : Whether the Conclufion drawn from them is plain and evident; and whether the Experiments on Pendulums, that confirm the Theory, are eafy to be made, and may be depended upon.

<div align="right">Tho'</div>

Tho' Sir *Isaac Newton*, in his *Principia*, has not endeavour'd to give the Cause of Gravity, or to determine whether it be owing to an Impulse or not ; yet he has shewn what its Effects and Laws are, from plain Experiments made by others and himself. From the Laws of Gravity, and from the Observation of a Comet, * he has deduced the Annual Motion of the Earth ; and it must have a Diurnal Motion, if it has an Annual one, otherwise, it will not agree with the *Phænomena*. The Laws of the Centrifugal Force, or that Force by which a Body, whirl'd round in any Circle, endeavours to recede from the Center of its Motion, have been demonstrated by Monf. *Huygens*.

Thefe are the Principles from which Sir *Isaac Newton* draws his Conclusion ; and tho' some Persons, that will not be at the Pains to examine them, may deny them by the Lump, yet no Body has yet been able to shew any Flaw in the Demonstrations that relate to them.

NB. *This Dissertation will be continued in our next.*
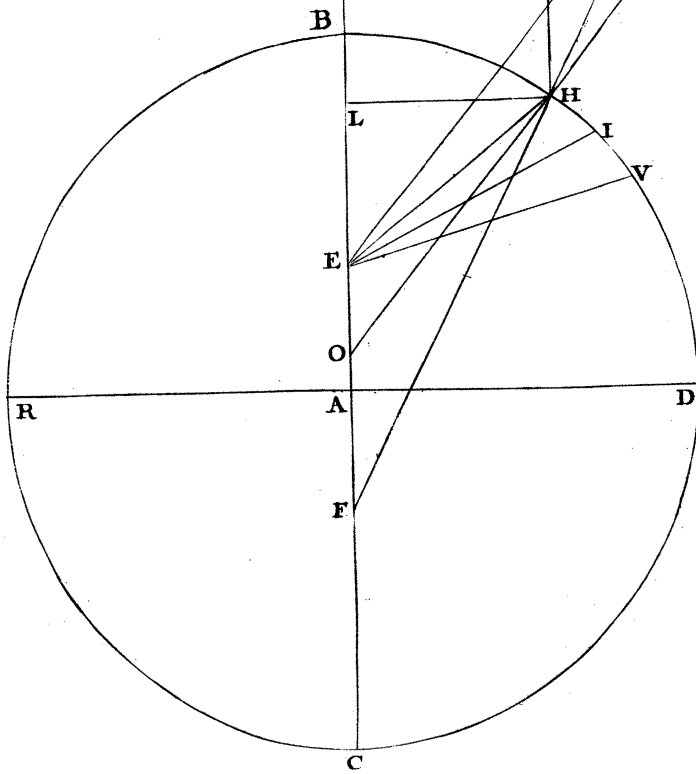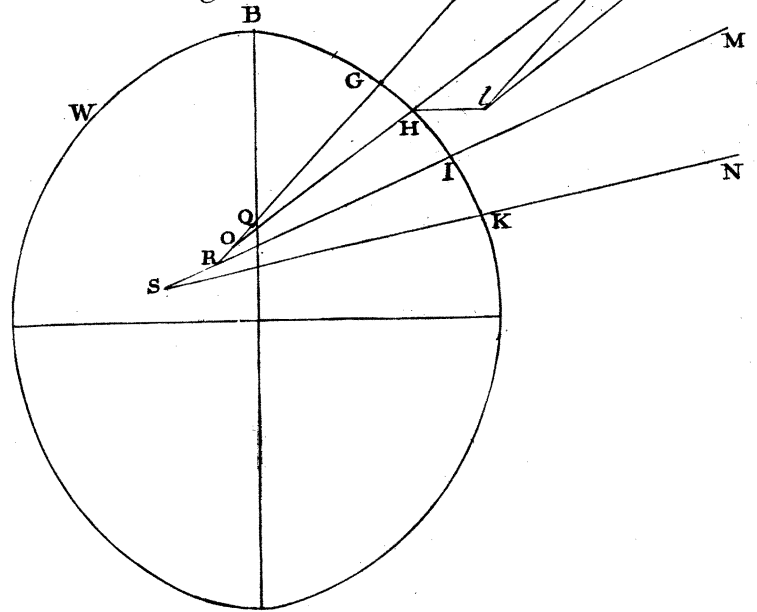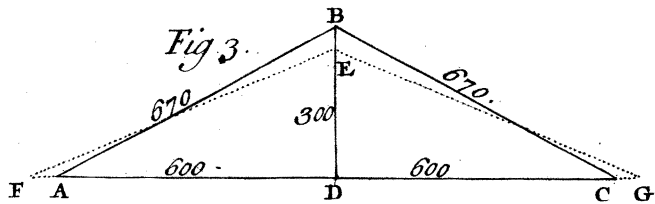
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* *Princip. Lib.* 3. *Prop.* 12, 13, & 42.

II. *Some*

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Parker *sculp.*